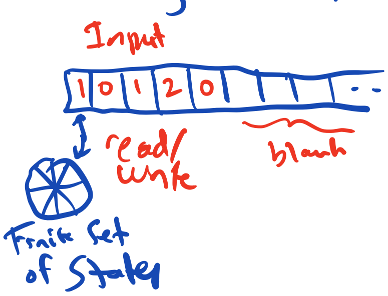
Lecture 3

CSE 431 Intro to Theory of Computation

Turing Machines



Q Finite set of states

Σ input alphabet (finite)

Γ tape alphabet (finite)

$\sqcup \in \Gamma \setminus \Sigma$

'blank'

$q_0 \in Q$ start state

$q_{acc} \in Q$ accept state

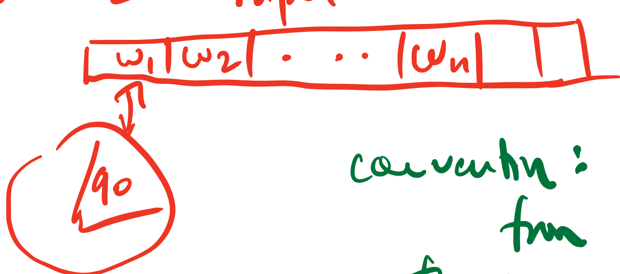
$q_{rej} \in Q$ reject state

Transition function $\delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

old state, scanned symbol, new state, new symbol, move left or right

Classroom MGH 058 starting Monday

$w \in \Sigma^*$ input $\quad |w| = n$



convention: if move from left end of the tape has an L just stay there.

eg $\{\underbrace{y\#y}_{\text{input}} : y \in \{0,1\}^*\}$ $\quad y$ binary string

clock

ø x ø x # ø x ø x.

remember ø

1

{#}

- Record & mark 1st character
- Scan right past #
- Scan to first unmarked char.
- if ≠ recorded char reject

  else move left past #

  move left until a marked char and move right 1 step

- if 1st char is a #

  check the rest is marked prior to first blank

- if yes accept

  no reject

Implementation level description

Formalizing computation

"Configuration"

snapshot of a TM on an input at some step

- current state
- tape contents
- head position of TM

$Q \cap \Gamma = \phi$

$\underbrace{\Gamma^*}_{\text{left}} Q \underbrace{\Gamma^*}_{\text{right}}$

Configuration $uqav$ $\quad u \in \Gamma^*$ $\; v \in \Gamma^*$
C $\quad a \in \Gamma$ $\; q \in Q$

currently scanned symbol

$\delta(q,a) = (p,b,R)$ $\qquad \vdash_M$ yielding one step

$$\frac{uqav \vdash_M ubpv}{\delta(q,a) = (p,b,L)}$$

$$qav \vdash_M pbv$$

$$\frac{ucqav \vdash_M uqcbv}{\delta(q,a) = (p,b,R)}$$

$$uqa \vdash_M ubq\sqcup$$

blank

Nothing from $uq_{acc}av$
$uq_{rej}av$

$C \vdash_M^* D$ yields in some number of steps

Start configuration on input w?

$q_0\ w$

Def M accepts w iff

$q_0 w \vdash_M^* u q_{acc} v$ for some u, v

M rejects w iff

$q_0 w \vdash_M^* u q_{rej} v$ for some u, v.

Defn $L(M) = \{ w \mid M \text{ accepts } w \}$
language recognized by M.

Defn M is a decider iff
for every $w \in \Sigma^*$
M accepts w or M rejects w

M decides L iff
L(M) = L and M is
a decider.

Notation for DFA

Mode for each state

$q \xrightarrow{a} p$

TM

$\delta(q,a) = (p,b,R)$

diagram

$q \xrightarrow{a \to b,R} p$

$\{0^{2^n} : n \geq 0\}$

0

00

0000

$0\ \cancel{0}\ 0\ \cancel{0}\ 0\ \cancel{0}\ 0\ \cancel{0}$

$\uparrow$

Scan to see if
exactly one 0
if so, accept

Defn A language L is Turing-recognizable iff there is a TM M s.t. $L = L(M)$

L is decidable iff there is a decider TM M s.t. $L = L(M)$